Activity description

This activity can be used as an introduction to differentiation. Students are shown how to use a spreadsheet to find the gradients of functions of the form x^n. This leads to the general rule for gradients of functions of this type.

Suitability

Level 3 (Advanced)

Time

2– 3 hours

Resources

Student information sheet, worksheet, spreadsheet.
Optional: slideshow

Equipment

Computer access with spreadsheet

Optional: graphic calculator or graph paper

Key mathematical language

Function, gradient, curve, tangent, negative gradient, positive gradient, gradient function, square, cube.

Notes on the activity

Slides 1– 6 of the slideshow can be used to introduce differentiation.
A shortened version is given on pages 1 and 2 of the student sheets.

Students can then use the spreadsheet to complete Section A using the first worksheet; this is set up to find the gradients of a series of chords from the point P(3, 9) on $y = x^2$. Fill-down will enable students to see the gradient approaching the value 6 as Q approaches P.

Students are asked to find the gradient at other points on the curve, and hence deduce the relationship between the gradient and the value of x.

Section B and the second worksheet can be used to find the corresponding result for $y = x^3$.

In Section C, after setting up their own worksheet for $y = x^4$, students are asked to look for a pattern in the gradient functions and to suggest the gradient function for $y = x^5$, $y = x^6$ and $y = x^n$.

Slide 7 can be used to summarise the results and to aid class discussion.
Slide 8 can be used to help students to reflect on the work they have done.

The rest of the slides are optional extras which you can use to extend the work or leave until a later date.

Slide 9 shows how the results for $y = x^2$ can be proved from first principles.

Slide 10 summarises the rules needed for differentiating polynomials.

Slide 11 shows how these rules are applied to a compound function.

The final slide illustrates the links between important features in the graphs of the original function and its derivative.

During the activity

Students can work individually or in pairs.

Points for discussion

Discuss the results for each of $y = x^2$, $y = x^3$ and $y = x^4$ with students.

The result for $y = x^2$ should be fairly obvious, but the patterns for the gradient functions of $y = x^3$ and higher powers may be less clear.

You could ask students to plot the results by hand or on another spreadsheet or graphic calculator to see the pattern more clearly.

The use of a trendline in Excel, or the function fitting facility on a graphic calculator, can help confirm the results.

Extensions

Students could make copies of the $y = x^2$ and $y = x^3$ worksheets and modify them to find the gradients of functions such as $y = 3x^2$ and $y = 2x^3$ to illustrate the rule for functions of the form $y = ax^n$.

The last two slides could be used to move on to more complex functions and the links between the main features in the graphs of a function and its derivative.

Answers

The teachers' spreadsheet contains the results when fill-down is used on the worksheets in the students' spreadsheet as well as graphs of $y = x^2$. These can be used to aid class discussion during the activity.

See the next page for completed versions of the tables on pages 3 and 4 of the students' sheets.

Answers

y

Point	x coordinate	Gradient
(–4, 16)	–4	–8
(–3, 9)	–3	–6
(–2, 4)	–2	–4
(–1, 1)	–1	–2
(0, 0)	0	0
(1, 1)	1	2
(2, 4)	2	4
(3, 9)	3	6
(4, 16)	4	8

Gradient = $\mathbf{2x}$

y

Point	x coordinate	Gradient
(–4, –64)	–4	48
(–3, –27)	–3	27
(–2, –8)	–2	12
(–1, –1)	–1	3
(0, 0)	0	0
(1, 1)	1	3
(2, 8)	2	12
(3, 27)	3	27
(4, 64)	4	48

Gradient function = $\mathbf{3x^2}$

y

Point	x coordinate	Gradient
(–4, 256)	–4	–256
(–3, 81)	–3	–108
(–2, 16)	–2	–32
(–1, 1)	–1	–4
(0, 0)	0	0
(1, 1)	1	4
(2, 16)	2	32
(3, 81)	3	108
(4, 256)	4	256

Gradient = $\mathbf{4x^3}$

Equation of curve	Gradient function
$y = x^2$	$2x$
$y = x^3$	$3x^2$
$y = x^4$	$4x^3$
$y = x^5$	